

Research and Solution of Laser Marking Based on NURBS Model

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Abstract: This paper mainly studies the laser marking hatch contour generation problems, by focusing on the mathematical model of the genetic algorithm was combined with Newton interpolation and NURBS curve, explore the hatch of given data of two kinds of outline design and program running time, the total length of curve and reasonable optimization, the laser marking efficient application in industrial practice to provide reasonable theory basis.

For problem 1, considering the required number and length of curves, many curve characteristics that can be expressed by monotone functions can be divided by using closed curves. By constructing NURBS curves and using Genetic algorithm model to infinitely approximate the curves, the isometric lines can be found out. Then use Newton interpolation method to select each adjacent three points and the corresponding quadratic function, using the quadratic function in the middle of the three points of derivative calculation after the middle point of normal, pointing in the direction inside the curve along the line again hatch out of step and satisfy the contour parallel algorithm under a curve, and the curve as the input of the next iteration, as long as the curve on any point pair of distance is less than the step length, the terminating, hatch contour parallel algorithm is complete; In addition, the curve fitted is smoothed and inflected by means of data enhancement, so as to avoid the problem of breaking point caused by too small curvature of extreme point of some curves. Finally, the time function is used to calculate the program running time ratio.

For question 2, the difference between question 1 and question 2 lies in the need to implement the algorithm of nesting multi-layer contour patterns. First, on the basis of question 1, the idea of classification discussion is mainly considered to divide various types of curves that can be represented by monotone function in a more detailed way. Then, the inner and outer arcs of curves can be judged more accurately and efficiently, so as to facilitate the direct contraction from the curve to the inside of the curve. Finally, the number of curves, the total length and the running time of the program are calculated by combining the modeling of problem one- and the-time function.

For question 3, the concavity of the curve is determined mainly through Lagrange mean value theorem, so as to optimize the process of judging the internal direction of the curve and the efficiency and quality of the program operation. First of all, considering that the CPU performance or memory capacity of the factory's computer in real life cannot support the complicated judgment conditions in the process of a large number of iterations, we choose Lagrange mean value theorem as the basis and adopt the idea of iterative hypothesis re-verification to optimize the determination method of the inside and outside of the curve. Iteration starts from the first three adjacent points A, B and C. Each time, it is assumed that... Point A is the starting point of the piecewise curve and point C is the end point of the piecewise curve. Then, the corresponding one-time functions of point A and Point C are calculated according to these two points, and the value and magnitude relation of point B and its function on the one-time function are recorded. Then, the next point D after point C is iterated. Finally, the linear function determined from two points A and D determines the relationship between point C and Point B and their values on the linear function to determine whether they are in the same monotone curve region. Repeat the above process repeatedly until all points are exhausted, so that the overall program can obtain the optimization effect.

1. Introduction

Laser is an important invention in the 20th century. It is characterized by accurate direction and easy control. On this basis, laser marking technology has become a key technology in industrial processing. It applies laser to the surface of processed products with high efficiency, no consumables, minimal impact on the surface deformation of products, and high firmness of the marking content. However, laser marking machine groove tool parameters, material and other factors of the selection of different processing efficiency and effect of its products have certain effects. Therefore, it is very important to choose suitable groove cutter parameters and material to improve the utility of industrial processing.

2. Assumptions

Hypothesis 1: Assume that the curve connected by the hatch groove is closed without breakpoint;

Hypothesis 2: Assume that the curve connected by the hatch groove is smooth;

Hypothesis 3: It is assumed that the material used for laser marking groove cutter is uniform and has no qualitative change.

Hypothesis 4: Hatch material is guide surface, suitable for laser marking.

3. Symbol Description.

Table 1. Symbol Description.

symbol	symbol description
$P(u)$	Plane parameter curve
$N(u)$	The unit normal vector at some point
$C(u)$	M NURBS curve
$C_0(u)$	Equidistant line
$S(u)$	Approximation curve
λ	Parameter

4. Establishment and Solution of the Model

4.1 Establishment of Problem 1 Model-Establishment of Genetic Algorithm Optimization Model

4.1.1 Isometric line construction based on NURBS curve algorithm and genetic algorithm model optimization.

A plane parameter curve is established, $P(u) = (x(u), y(u))$, u is an argument whose distance is the equidistant curve $P_{off}(u) = P(u) \pm d \cdot N(u)$, One when the curve outward vector direction of positive (negative) isometric when using “+”(“-”), $N(u)$ is the curve $P(u)$ at point $(x(u), y(u))$ the unit normal vector of: $N(u) = \left(\frac{-\dot{y}(u)}{\sqrt{\dot{x}^2 + \dot{y}^2}}, \frac{\dot{x}(u)}{\sqrt{\dot{x}^2 + \dot{y}^2}} \right)$, where $\dot{x}(u)$ and $\dot{y}(u)$ respectively for $x(u)$ and $y(u)$ of a derivative. Next, genetic algorithm is used to construct NURBS isometric lines through parameter optimization.

The basic process of equidistant line construction is as follows:

Step1. Determine the constraint condition formula of the always point and the adjacent control vertex according to the continuity condition at the endpoint;

Step2. According to the curvature of the original curve, calculate a batch of accurate isometric points according to the precision of 1×10^{-12} two more decimal places of the data;

Step3. Use Newton's interpolation method to fit the calculated exact isometric points to obtain isometric lines;

Step4. Use parameter optimization method to calculate the modified parameter value of accurate isometric point, and use the new parameter value to re-fit isometric line;

Step5. Error estimation. For points that do not meet the accuracy, insert the heavy node and turn to Step3 to reconstruct the isometric line until the accuracy is met.

Given that m-order NURBS curve is defined as:

$$C(u) = \frac{\sum_{i=0}^n N_i(u)w_i P_i}{\sum_{i=0}^n N_i(u)w_i}, u \in [u_m, u_{m+1}] \quad (1)$$

According to the above formula isometric line is:

$$C_0(u) = C(u) + d \cdot N(u), u \in [u_m, u_{m+1}] \quad (2)$$

Now construct an M-order NURBS curve $S(u)$ to approximate the curve $C_0(u)$:

$$S(u) = \frac{\sum_{i=0}^n N_i(u)w_i S_i}{\sum_{i=0}^n N_i(u)w_i}, u \in [u_m, u_{m+1}] \quad (3)$$

From the continuity condition:

$$\begin{cases} S_0 = P_0 + d \cdot N(u_m) \\ S_{n_1} = P_n + d \cdot N(u_{n+1}) \\ S_1 = s_0 + \lambda_1(P_1 - P_0) \\ S_{n_1-1} = S_{n_1} + \lambda_2(P_{n+1} - P_n) \end{cases} \quad (4)$$

Where λ_1, λ_2 is the constant to be solved?

According to the precision requirements, $C_0(u)$ is discretized, and the approximation error of the original curve of the point series obtained after discretization is estimated by the osculating circle. Using parameter optimization method to improve the global fitting of these discrete point series $P_i (i = 0, 1, \dots, k)$. After the optimal fitting, the non-optimal points (some other points outside the sampling point) should be estimated. That is to take a number of points on $C(u)$ and examine the local minimum distance from these points to $S(u)$. If d_1 is greater than the specified error, the procedure is repeated by inserting some new nodes into the corresponding node interval.

The entire NURBS curve approximation process can be represented by a flow chart:

Figure 2 a flow chart of NURBS equidistant curve construction

4.1.2 Newton interpolation method to fit the curve. Based on the above model and combined with Newton interpolation method, contour parallel hatching algorithm is realized:

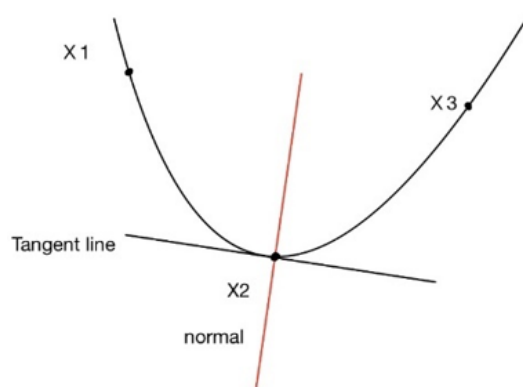


Figure 1. Tangent, normal diagram.

Using closed curve can be divided into a finite number can be expressed in monotone function curve of this trait, combined with the Newton interpolation method to each division out of curve fitting,

namely each adjacent three points x_1, x_2, x_3 using Newton interpolation method of corresponding quadratic function $y = a \times x^2 + b \times x + c$, and then use the quadratic function in the middle of the three points of derivative, namely $f'(x) = 2 \times a \times x + b$, to work out after the middle point of normal slope $-\frac{1}{dy}$.

Then walk out the step length along the direction that the method points to the inside of the curve, and work out the next curve that satisfies the contour parallel incubation algorithm, and make the data points on the curve as the input data of the next iteration. Finally, as long as the distance between any pair of points on the curve is less than the step length, the iteration is terminated and the contour parallel incubation algorithm is completed.

4.1.3 Data enhancement method for smoothing. Handling methods when smooth and inflection points are encountered.

There are several types of inflection points.



Figure 2. igzag inflection point. Figure 3. ight Angle inflection. Figure 4. Z' inflection point.

In the segmentation proces, although the closed curve can be divided into several curves that can be expressed by monotone function, some extreme points in the curve have too little curvature, and even form a tip, which makes it difficult for Newton interpolation method to make the fitted curves of adjacent regions intersect.

To handle several disjoint curve, the use of an area by increasing the amount of data to discrete into continuous thoughts, thus through data enhancement method, namely using extremum point within the neighborhood of each pair of adjacent points to calculate straight slope, the equivalent of two monotone curve in each area, then add an area, and then use the iterative generator according to the slope generated a number of points, in the process of generation, to be enhanced sample data, then a new generation of counted into the next iteration of the input, can effectively solve the problem of curve is not smooth or a breakpoint.

4.2 Solution to problem 1.

First, use the data in Graph1 to draw the image, as shown in the figure below.

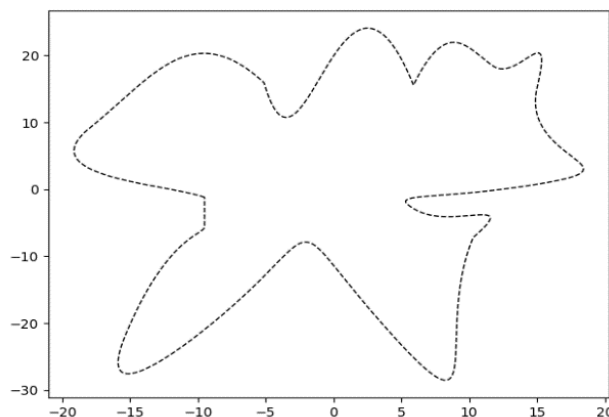


Figure 5. Laser labeled hatch outline.

According to the shape of the image, find out the corresponding extremum point, and then divide into 6 regions according to the extremum point, and carry out fitting respectively.

Since the data in Graph1 was given according to the extension direction of the curve, all the extreme points of X and y could be found by iterating twice according to the change rule of X and Y. The following table shows the horizontal and vertical coordinates of the 16 extreme points:

Table 2. The horizontal and vertical coordinates of an extremum.

X	Y	X	Y
-13.95072281	0.733057681362212	14.9210611558164	12.00479837
-9.090901434	20.2624658536613	18.3754048508972	2.725028217
-3.30361602	10.7796755805435	5.35010308896504	-2.02312713
2.76363077968873	24.02365684	11.5306106872098	-4.536600416
6.42194260050333	17.58181811	8.1218042102098	-28.5101303
9.10311185164155	21.90810703	-2.266547839	-7.9412357479
12.6665248204405	18.01299887	15.1854770786086	20.30608277

The six segmented regions are shown as follows:

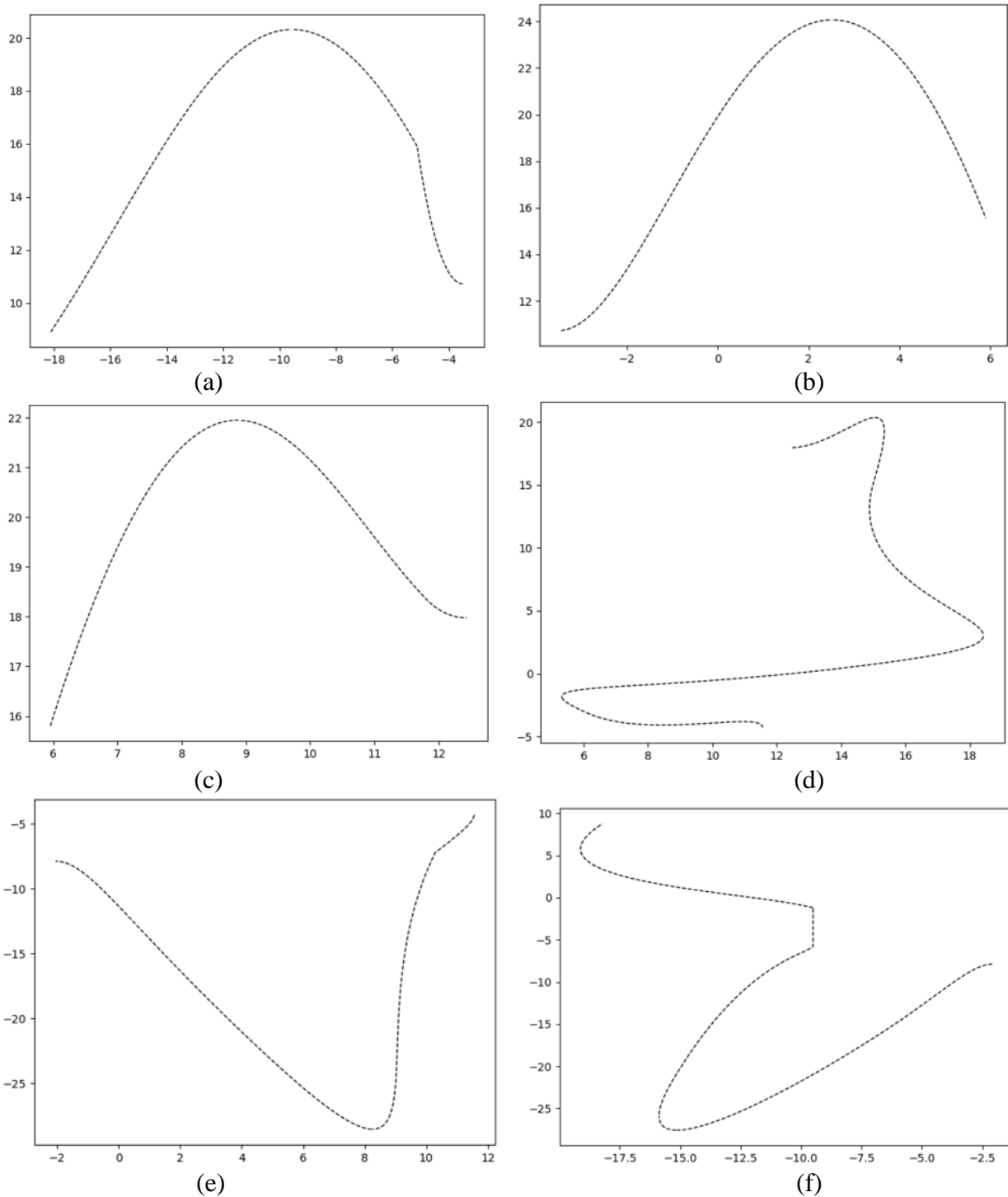


Figure 6. (a, b, c, d, e, f) Regional segmentation diagram.

In the process of fitting, the inflexion points as shown in the figure below is encountered, which conforms to several inflexion points described in 1.1 model establishment. The method of data enhancement mentioned above is used to optimize the inflexion point, and the fitting is continued after the inflexion is optimized.

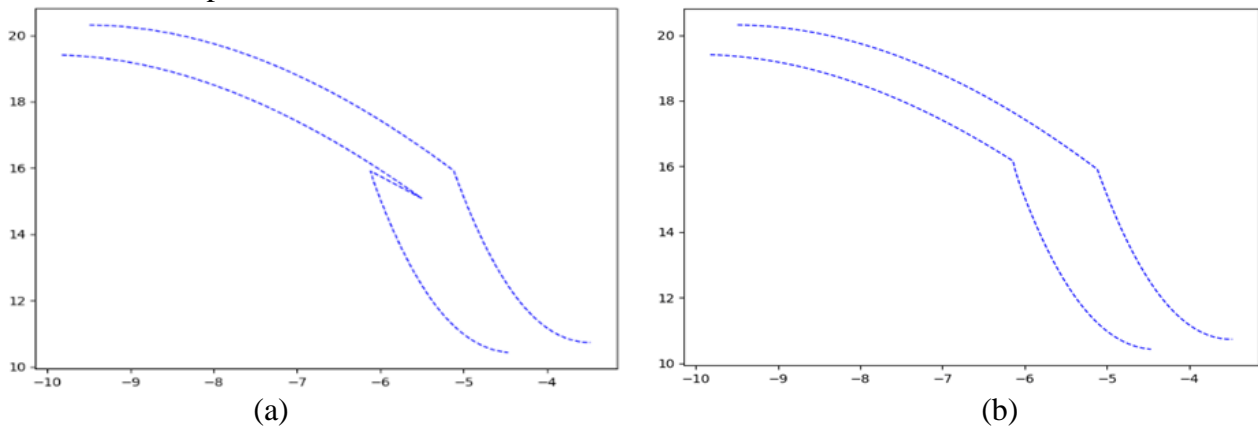


Figure 7. Graph of inflexion points encountered.

According to the title, the two groups of parameters are respectively:

- (1) Internal shrinkage boundary distance is 1mm, hatch spacing is 1mm
- (2) Internal shrinkage boundary distance 0.1mm, hatch spacing 0.1mm

1. Solving the number of parallel lines and running time of sawtooth parallel hatches under **two** parameters.

The parameter is the step length. According to the given step length, the established model is used to determine the isometric point, and then the NURBS isometric line is fitted, and the above process is repeated (see Figure 1 A flow chart of NURBS equidistant curve construction), and multi-layer parallel lines under two groups of parameters are obtained respectively.

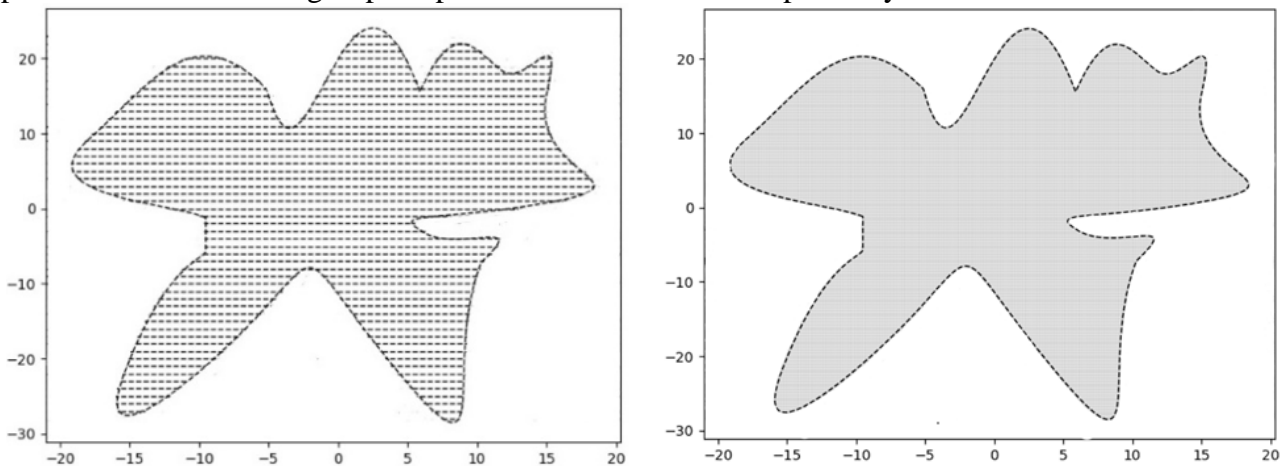


Figure 8. Hatch outline drawing under two sets of parameters.

According to the maximum, minimum and step size of the curve, the following results can be obtained:

(1) When the step length is 1mm, the number of contour coils = 53, and the average running time of incubation program = 0.03391003608703613 ms, the total length = 1451.2128318212843mm.

(2) When the step length is 0.1mm, the number of contour coils = 527 and the average running time of incubation program = 0.32114148139953613 ms, the total length = 13341.2316221784813 mm.

The running time ratio of the program under the conditions of parameter group 2 and group 1 = 9.47039633267477.

2. The solution of contour winding number and running time for contour parallel hatch **under** two kinds of parameters.

In the same way as in the first case, the multi-layer curve contour parallel graphs under two groups of parameters are obtained respectively.

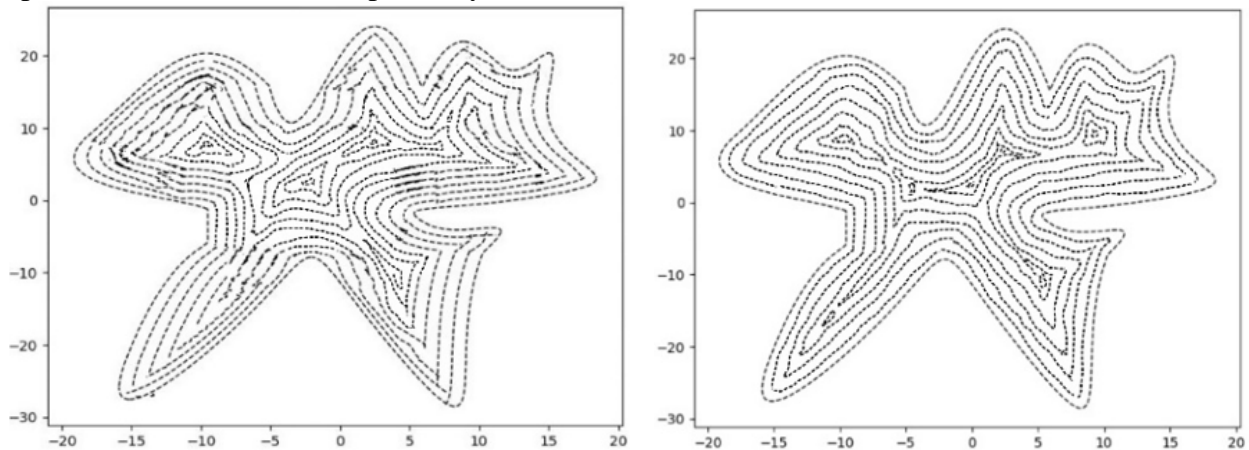


Figure 9. Multilayer curve contour parallel diagram.

According to the maximum, minimum and step size of the curve, the following results can be obtained:

(1) When the step length is 1mm, the number of contour coils = 18, and the average running time of incubation program = 0.03298508167266845ms the total length = 912.7123211287442mm.

(2) When the step length is 0.1mm, the number of contour coils = 171, and the average running time of incubation program = 0.42985081672668457ms, the total length = 9012.2218328246462mm.

The running time ratio of the program under the conditions of parameter group 2 and group 1 = 13.031673560561787.

4.3 The Establishment and Solution of Problem Two Model

4.3.1 Establishment of Problem 2 Model

This question is carried out on the basis of the first question. The model and algorithm flow used are the same as the first question, which will not be repeated here. Instead, the solution will be focused on the basis of the difference from the first question.

4.3.2 Solution to problem 2.

Based on the data in Graph2, the outline was drawn and the solution was carried out on the basis of the established model and algorithm.

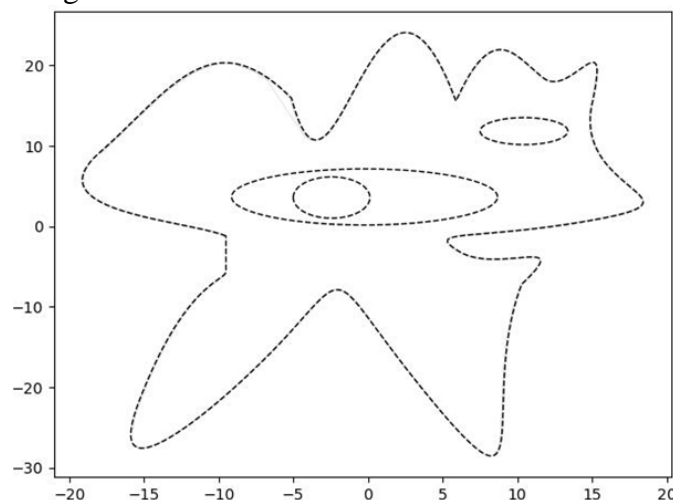


Figure 10. Laser labeled hatch outline 2.

(1) The solution of the number of parallel lines and running time of sawtooth parallel hatches under two parameters.

Based on the given step length, the established model is used to determine the isometric points, and then the NURBS isometric lines are fitted, and the above process is repeated (see the isometric curve construction flow chart in FIG. 1 for the detailed process), and the multi-layer curve contour parallel diagrams under two groups of parameters are obtained respectively.

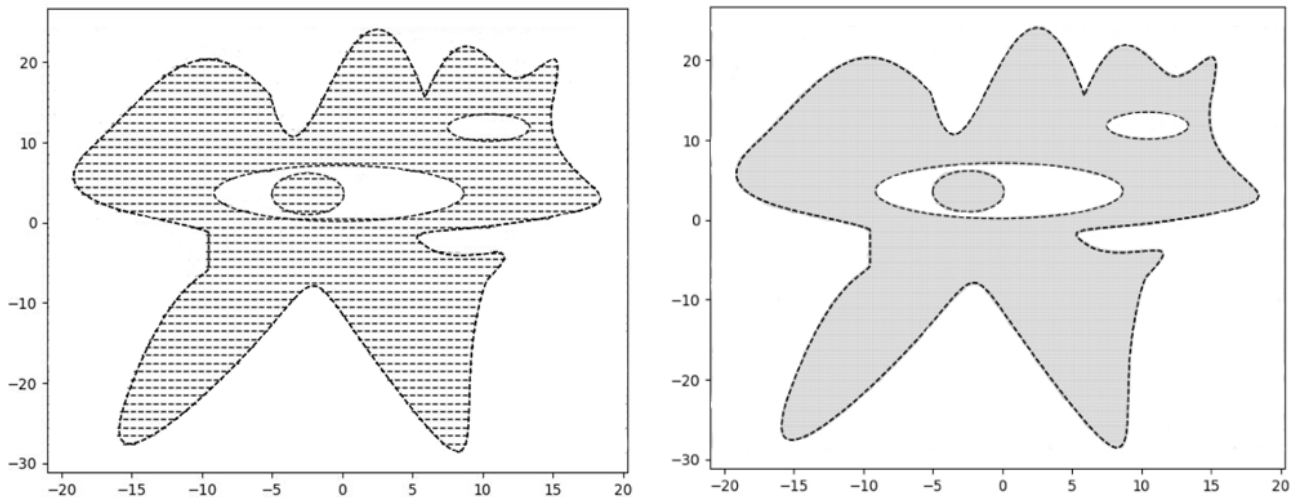


Figure 11. Laser labeled hatch outline 2.

With problems on the basis of a split in the same area, and ruled out a few middle areas, minimum and maximum in all areas of the scan every time step of 1 mm and 0.1 mm respectively answer respectively, due to the density of the curve is too dense, and adjacent point distance basic negligible, so each the length of the curve can be regarded as the sum of all adjacent points of distance, the results are as follows:

1) When the step length is 1mm, the number of contour coils = 62, the average running time of incubation program = 0.03791002208703613ms, and the total length = 721.9912973281mm;

2) When the step length is 0.1mm, the number of contour coils = 609, the average running time of incubation program = 0.34112748132353629ms, and the total length = 7880.1217224822mm;

The running time ratio of the program under the conditions of parameter group 2 and group 1 = 8.998345623232693

(2) The solution of the number of parallel lines and running time of contour parallel hatches under two parameters.

According to the trend of monotone function, all curves are divided into four types: R_1 , R_2 , R_3 and R_4 which are specifically defined as follows:

1) Clockwise:

The slope of the tangent line at all points on R_1 curve is $k > 0$, increasing monotonically;

The slope of the tangent line at all points on R_2 curve is $k < 0$, decreasing monotonically;

The slope of the tangent line at all points on R_3 curve is $k < 0$, increasing monotonically;

The slope of the tangent line at all points on R_4 curve is $k > 0$, decreasing monotonically.

2) Counterclockwise direction:

The slope of the tangent line at all points on R_1 curve is $k > 0$, decreasing monotonically;

The slope of the tangent line at all points on R_2 curve is $k < 0$, increasing monotonically;

The slope of the tangent line at all points on R_3 curve is $k < 0$, decreasing monotonically; the slope of the tangent line at all points on R_4 curve is $k > 0$, increasing monotonically.

3) General conditions:

The slope of the tangent line at all points on R_1 curve is $k > 0$, increasing monotonically;

The slope of the tangent line at all points on R_2 curve is $k < 0$, decreasing monotonically;

The slope of the tangent line at all points on R_3 curve is $k < 0$, decreasing monotonically; the slope of the tangent line at all points on R_4 curve is $k > 0$, increasing monotonically.

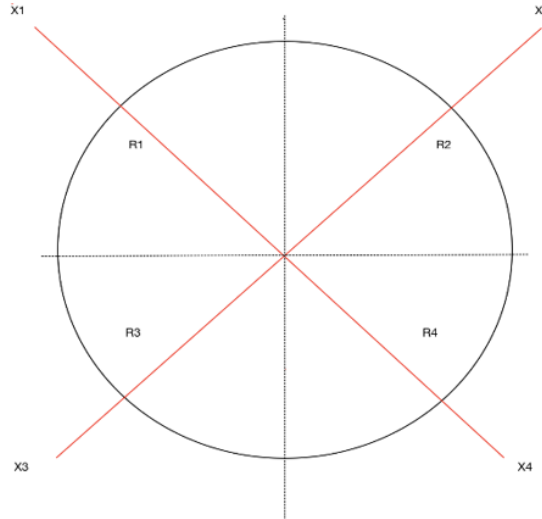


Figure 12. Take circles as an example to divide four kinds of curves.

Then, according to the above four types of curves for judgment, then according to the relative to all points of extreme value point of ordinate y algorithm process, the specific process was about three Newton interpolation method of quadratic function fitting x guide into a function, get the middle point of the tangent slope k , l for contour step length (1 mm or 0.1 mm), yy for contour line contraction after point ordinate, xx as abscissa point, after four kinds of situations:

(1) When the fitting curves are of type I, define $rad1$ ():

$$yy = y - l * \text{abs}(\sin(\text{atan}(k)))$$

$$xx = x + l * \text{abs}(\cos(\text{atan}(k)))$$

(2) When the fitting curves are of type II, define $rad2$ ():

$$yy = y - l * \text{abs}(\sin(\text{atan}(k)))$$

$$xx = x - l * \text{abs}(\cos(\text{atan}(k)))$$

(3) When the fitting curves are of type III, define $rad3$ ():

$$yy = y + l * \text{abs}(\sin(\text{atan}(k)))$$

$$xx = x + l * \text{abs}(\cos(\text{atan}(k)))$$

(4) When the fitting curves are of type IV, define $rad4$ ():

$$yy = y + l * \text{abs}(\sin(\text{atan}(k)))$$

$$xx = x - l * \text{abs}(\cos(\text{atan}(k)))$$

This is the operation process of the extreme point of y relative to all points. When the extreme point of x relative to all points is operated, all plus signs become minus signs, and minus signs become plus signs, then the multiplexing of the algorithm can be completed.

The results are as follows:

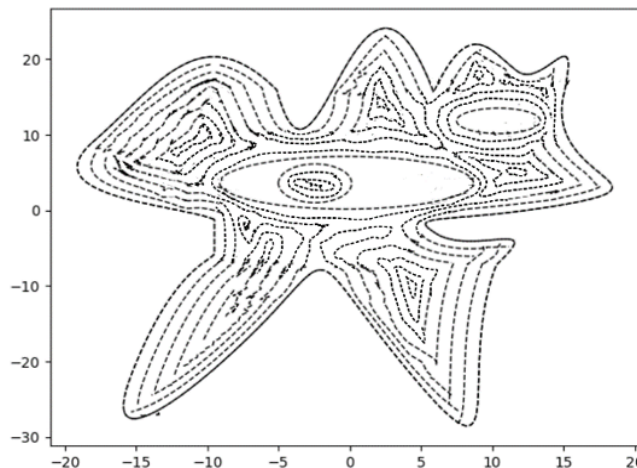


Figure 13. Hatch outline drawing under two sets of parameters2.

According to the maximum, minimum and step size of the curve, the following results can be obtained:

1) When the step length is 1mm, the number of contour coils = 14, the average running time of incubation program = 0.030974111672062ms, and the total length = 879.0657911208236mm;

2) When the step length is 0.1mm, the number of contour coils = 152, the average running time of incubation program = 0.40922081612663238ms, and the total length = 8520.1237216332645mm;
The running time ratio of the program under the conditions of parameter group 2 and group 1 = 13.211704679677416

4.4 The Establishment and Solution of Problem Three Model.

4.4.1 Establishment of Problem 3 Model.

According to Lagrange's mean value theorem, the theorem states that if the function $F(x)$ satisfies, is continuous on the closed interval $[A, b]$ and differentiable in the open interval (A, b) , then it can be concluded that there is at least one point in the open interval (A, b) , so that the following equation is true:

$$f(b) - f(a) = f'(\varepsilon)(b - a) \quad (5)$$

In addition to:

$$\varepsilon = a + \theta(b - a) \quad (0 < \theta < 1), \quad a = x, b = x + \Delta x, \quad (6)$$

Then you can derive:

$$\Delta y = f(x + \Delta x) - f(x) = f'(x + \theta \Delta x) \cdot \Delta x \quad (0 < \theta < 1) \quad (7)$$

$$dy = f'(x) \Delta x$$

Lagrange's mean value theorem realizes that when the independent variable x changes to $|\Delta x|$ is not very small, the exact expression of the function Δy can be obtained, which brings great convenience to the determination of the inside and outside of the arc of the curve

4.4.2 Solution to problem 3. To optimize the inspection.

Now the convexity of the curve is proved by Lagrange mean value theorem:

There are three adjacent points, x_0, x_1 and x_2 , which satisfy $x_2 > x_1 > x_0$. According to the Lagrange formula,

Under the following conditions:

$$\begin{aligned} 0 < \theta_1 < 1 \quad 0 < \theta_2 < 1 \\ x_0 < x_1 < x_2 \quad x_1 - x_2 = h \quad x_1 - x_0 = h \end{aligned} \quad (8)$$

According to the Lagrange mean value theorem, it can be concluded that:

$$f(x_2) - f(x_1) = f'(x_1 + \theta_1 h)h \quad (9)$$

$$f(x_1) - f(x_0) = f'(x_0 + \theta_2 h)h \quad (10)$$

Subtract (9) from (10) to get (11):

$$f(x_2) + f(x_0) - 2f(x_1) = [f'(x_1 + \theta_1 h) - f'(x_0 + \theta_2 h)]h \quad (11)$$

(3) According to Lagrange's mean value theorem, it can be deduced that:

$$\begin{aligned} [f'(x_1 + \theta_1 h) - f'(x_0 + \theta_2 h)]h &= f''(\varepsilon)(\theta_1 + \theta_2)h^2 \quad x_0 + \theta_2 h < \varepsilon < x_1 + \theta_1 h \\ f''(\varepsilon) &> 0 \\ \frac{f(x_1 + h) + f(x_1 - h)}{2} &> f(x_1) \\ \frac{f(x_0) + f(x_2)}{2} &> f\left(\frac{x_0 + x_2}{2}\right) \end{aligned} \quad (12)$$

It is proved that the curve is concave, which corresponds to R1 and R4 in problem 2. If (4) results are opposite and the results are conversely, the curve is proved to be convex, which corresponds to R2 and R3 in problem 2.

When the data point density is large enough and the sample data is large enough, the distance of h above is equivalent to approaching 0, that is, even if the distance between two pairs of points is different, the h of the two pairs of points can be considered equal. Then the convexity of the curve can be extended to the data points with different distances based on the Lagrange mean value theorem. In order, extend to the n point iterative process, with Lagrange theorem as the basis, from the very beginning the adjacent x_0 3 points, x_1, x_2 PM iterations, each hypothesis x_0 points for piecewise curve starting point, x_2 for piecewise curve the end, through the two point calculate x_0, x_2 corresponding function at a time, two points will be x_2 point and that point in a function to record on the basis of the size of the function values y_2 , in iterative x_2 points after a x_3 , through x_2, x_3 determine two points out of a function, To determine whether x_2 and x_1 are in the same region of a monotone curve by determining the magnitude of their values y_1 on the first function, the above procedure is repeated repeatedly until a point x_n has a magnitude relationship contrary to the recorded x_{n-1} , then x_{n-1} is proved to be the starting point of the next piecewise curve.

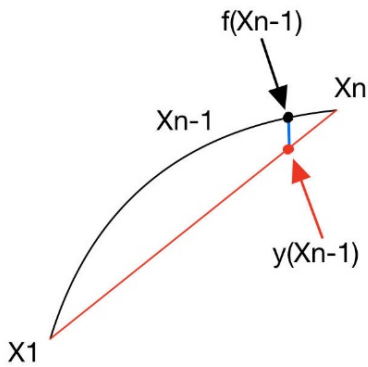


Figure 14. Class I curves.

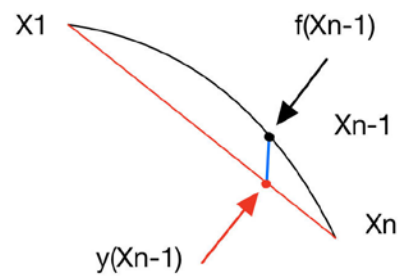


Figure 15. Class II curves.

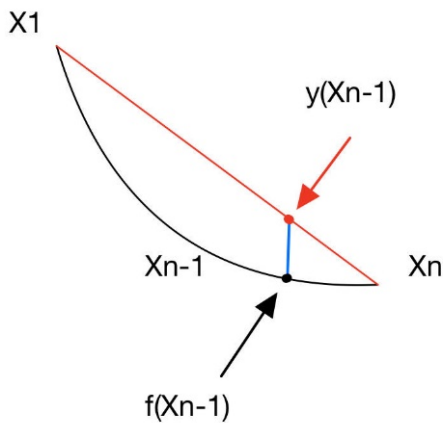


Figure 16. Class III curves.

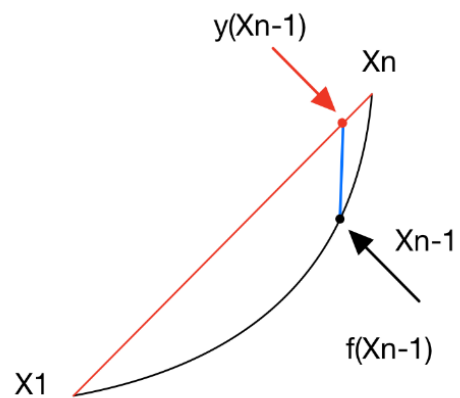


Figure 17. Class IV curves.

Through the successive iteration process, when the total number of samples is N , the entire the complexity of the algorithm will achieve linear level $O(N)$, storage space is $O(N)$, will be better than the first two ask judgment process, and all the terminating conditions need all the points on the curve of each iteration, and iterative N times, complexity is $O(N) \times O(N) = O(N^2)$, improve an order of magnitude, it will be for algorithm optimization and in industry should be possible.

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